Phys 410
Fall 2014
Lecture #7 Summary
23 September, 2014

We considered two-particle conservative-force interactions and found that the potential function $U(\vec{r}_1 - \vec{r}_2)$ does double duty by representing two forces: $\vec{F}_{12}$ and $\vec{F}_{21}$. In other words, one can recover the forces of interaction between the two particles as $\vec{F}_{12} = -\vec{\nabla}_1 U(\vec{r}_1 - \vec{r}_2)$ and $\vec{F}_{21} = -\vec{\nabla}_2 U(\vec{r}_1 - \vec{r}_2)$, where the subscripts on the gradients denote derivatives with respect to the coordinates of one of the particles. In addition, this single potential energy function properly keeps track of the total potential energy of the two particles interacting by means of the associated conservative forces. The total mechanical energy of the two-particle system is given by $E = T_1 + T_2 + U(\vec{r}_1 - \vec{r}_2)$, and this quantity is conserved.

This treatment can be generalized to an arbitrary number of particles interacting with each other by means of conservative forces, and under the influence of external conservative forces. The potential energy due to conservative-force interactions among an N-particle system is

$$U^{int} = \sum_{\alpha=1}^{\alpha=N} \sum_{\beta>\alpha}^{\beta=N} U_{\alpha\beta}(\vec{r}_\alpha - \vec{r}_\beta) .$$

We began to discuss the physics of oscillations. Any system with a minimum in the potential energy landscape $U(x)$ can have small harmonic oscillations around the minimum in the potential. Therefore, Hooke's law is a useful approximation for a dynamical system around a stable equilibrium. The Hooke's law spring constant is just the local curvature of the potential at the minimum (x = x_0): $k = d^2 U / dx^2 \big|_{x_0}$. A one-dimensional harmonic oscillator of mass m obeys the equation $m\ddot{x} = -kx$, where x is the displacement from equilibrium. Dividing through by m and defining the natural oscillation frequency $\omega_0 = \sqrt{k/m}$, the equation becomes $\ddot{x} = -\omega_0^2 x$. This equation can be solved in numerous ways, and the solutions can be written in several canonical forms, including:

1) $x(t) = C_1 e^{i\omega t} + C_2 e^{-i\omega t}$
2) $x(t) = B_1 \cos(\omega t) + B_2 \sin(\omega t)$
3) $x(t) = A\cos(\omega t - \delta)$
4) $x(t) = \text{Re}[Ae^{i(\omega t - \delta)}]$

All of these forms can be related to each other, as you will prove in homework. Note that each form has two unknown parameters, and they can be determined by two initial conditions.

We also considered the energy in simple harmonic motion. The total mechanical energy is $E = T + U = (m/2)\dot{x}^2 + (k/2)x^2$. Using form 3 above (for example), this can be written as $E = (k/2)A^2$, which is constant. The kinetic and potential energies are both varying with time as $\sin^2(\omega t - \delta)$ and $\cos^2(\omega t - \delta)$, respectively. They both oscillate between 0 and E twice per period of oscillation, and are exactly 180° out of phase. The shuttling of energy back and forth between two different forms (in this case potential and kinetic) is a hallmark of simple harmonic oscillation, and resonance.